Evidence of Chaotic Hierarchy in a Semiconductor Experiment

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We study the cooperative spatio-temporal behavior of semiconductor breakdown via both probabilistic and dynamical characterization methods (fractal dimensions, entropies, Lyapunov exponents, and the corresponding scaling functions). Agreement between the results obtained from the different numerical concepts (e.g., verification of the Kaplan-Yorke conjecture and the Newhouse-Ruelle-Takens theorem) gives a self-consistent picture of the physical situation investigated. As a consequence, the affirmed chaotic hierarchy of generalized horseshoe-type strange attractors may be ascribed to weak nonlinear coupling between competing localized oscillation centers intrinsic to the present semiconductor system.

Key words: Semiconductor breakdown, Nonlinear transport phenomena, Impurity impact ionization, Numerical analysis, Chaotic hierarchy.

In the study of semiconductor electronic breakdown we observe the self-generated formation of both spatial and temporal dissipative structures, when a biased voltage is applied at low temperatures [1]. The underlying nonlinear physics reveals critical phase transition behavior by varying the temperature at constant voltage [2]. There is an additional control parameter, the magnetic field, that changes the dynamics of this system [3]. In the following, we study the abrupt structural change of chaotic behavior in an exemplary semiconductor experiment by varying the magnetic field B at constant voltage V and temperature T. Schematically, the complex nonlinear behavior of the current flow I in extrinsic germanium at liquid-helium temperature can be shown as in Figure 1 a). Above a certain threshold V_{th}, there are periodic islands (shaded) in the V-B parameter space within a sea of chaos. In order to look for the ladder towards higher orders of chaos, recently proposed by Rössler [4] as chaotic hierarchy for dissipative nonlinear dynamical systems of at least four state variables, we operate under slight variation of the applied magnetic field, because this control parameter is less sensitive than the bias voltage.

Our experimental system consists of single-crystalline p-doped germanium, electrically driven into low-temperature avalanche breakdown via impurity impact ionization [1]. The sample geometry and the electronic measuring configuration are sketched in Fig. 1 b). With dimensions of about $(0.25 \times 2 \times 5)$ mm^3 and an impurity acceptor concentration of about 10^{14} cm^{-3}, the extrinsic germanium crystal carries evaporated ohmic aluminum contacts as indicated by the shaded areas. To provide the outer ohmic contacts with an electric field (voltage V), a d.c. biased voltage V_0 was applied to the series combination of the sample and the load resistor R_L. A d.c. magnetic field B perpendicular to the broad sample surfaces could also be applied using a superconducting solenoid surrounding the semiconductor sample. The resulting current I was obtained from the voltage drop at the load resistor. The inner probe contacts (of about 0.2 mm diameter) served for monitoring independently the partial voltages V_i ($i = 1, 2, 3$) along the sample. During the experiments, the semiconductor sample was always kept at liquid-helium temperature ($T = 4.2$ K) and

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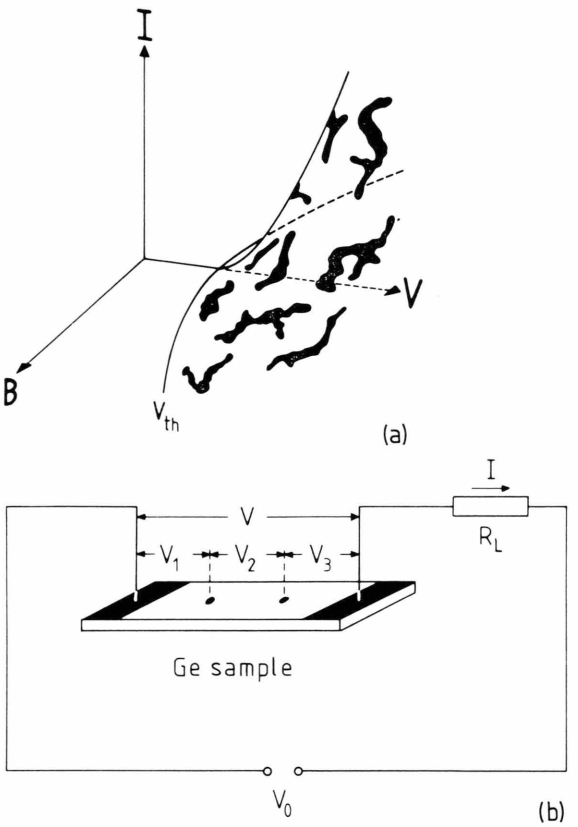

Fig. 1. Scheme of the semiconductor experiment: (a) parameter space, (b) experimental set-up.

carefully protected against external electromagnetic irradiation (visible, far infrared).

The complex spatial behavior of our semiconductor system can be globally visualized by means of low-temperature scanning electron microscopy [5]. As reported elsewhere [6] in detail, nucleation and growth of filamentary current patterns in the nonlinear post-breakdown regime are often accompanied by abrupt changes between different stable filament configurations via noisy current instabilities. Moreover, the simultaneous spatial identification of oscillatory current flow dynamics [7] provides a powerful tool for gaining deeper insight into the mutual interplay between spatial and temporal current structures. In the spirit of chaotic hierarchy [4], turbulent dynamics may thus be ascribed to nonlinear coupling between competing localized oscillation centers intrinsic to our semiconductor system. So far, we have demonstrated experimentally the existence of spatially separated oscillatory subsystems [8] as well as their long-range interaction [9].

In this experiment, we concentrate on the quantitative characterization of the cooperative temporal behavior induced by the avalanche breakdown kinetics of our multicomponent semiconductor system. For evaluating the hierarchical tree of the chaotic order proposed, we apply distinct numerical analysis procedures embracing both probabilistic and dynamical concepts. As a first step, we examine two characteristic data files of spontaneous voltage oscillations V_2 (t_n; $n=1, \ldots, 80\,000$), obtained for the different working conditions $B=31.5$ G (file A) and $B=46.5$ G (file B) at constant parameters $V_0=2.145$ V, $R_L=100\,\Omega$, and $T=4.2$ K. These cases were selected taking into account the different stuctural shape of the phase portraits shown in Figure 2. The two-dimensional representation $V_2(t_n)$ vs. $V_2(t_n+\tau)$ of the trajectories in phase space is constructed by using an appropriate sampling rate of 100 kHz and a delay time of 50 µs (embedding theorem [10]). As already pointed out in an earlier conjecture [3], the phase portrait of Fig. 2 a) is suggestive of a strange attractor having a dimension >2. The attractor can be visualized as a curled band partly folded over, embedded in three-dimensional space. This impression was especially striking when the bias voltage was slightly varied in the 0.1 percent range, resulting in a different projection of the same object. Upon increasing the magnetic field, the curled band structure (Fig. 2a)) gradually changed into a spherical tangle (Fig. 2b)) with increasing attractor dimensionality, apparently representing a higher state of chaos. The trajectories occupy the interior of a nearly spherical portion of the projected phase space. This picture did not change under small variations of the control parameters (cf. Fig. 3 of [3]).

In the following, we briefly report the quantification of the present experimental situation by the help of generalized fractal dimensions, entropies, Lyapunov exponents, and the corresponding scaling functions. The fundamentals of the characterization methods applied are described elsewhere [11] in detail. First, the generalized fractal dimensions $D(q)$ and the generalized entropies $K(q)$ were calculated with the nearest-neighbor algorithm proposed by Badii and Politi [12]. Taking into account the scaling behavior of the next-neighbor distance at a generic point with the number of trial points, the dimensions and entropies could be extracted directly from the slope and the shift of the successive log-log plots, respectively, obtained with

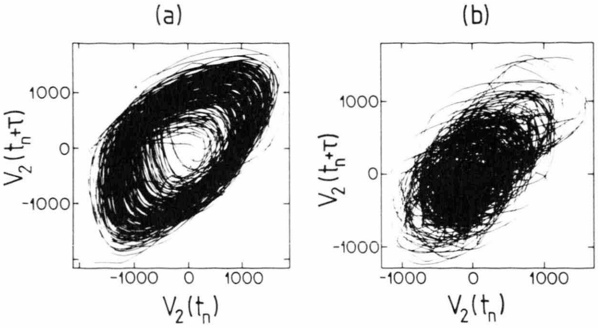

Fig. 2. Phase plots of different chaotic attractors generated from parts of the data files A (a) and B (b). Note that 1000 arbitrary units correspond to about 5 mV signal amplitude. The characteristic frequencies are below 5 kHz.

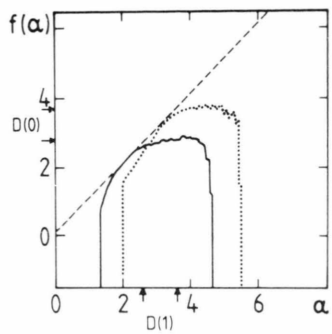

Fig. 3. Static scaling functions of different chaotic attractors calculated from the data files A (solid curve) and B (dotted curve). Note that the values of $D(0)$ and $D(1)$ indicated by arrows on the ordinate and the abscissa correspond to the maximum and the tangential point with the diagonal of the spectrum of dimensions, respectively.

Table 1. Comparison of characteristic quantities for different chaotic states. The values of the entropies and the Lyapunov exponents are in units of the sampling rate.

	File A	File B
Fractal Dimensions	$D(0)=2.6\pm0.1$ $D(1)=2.5\pm0.1$	$D(0)=3.6\pm0.1$ $D(1)=3.5\pm0.1$
Entropies	$K(0)=0.09\pm0.01$ $K(1)=0.09\pm0.01$	$K(0)=0.15\pm0.01$ $K(1)=0.15\pm0.01$
Lyapunov Exponents	$\lambda_1=0.095\pm0.005$ $\lambda_2=0.003\pm0.005$ $\lambda_3=-0.72\pm0.02$	$\lambda_1=0.159\pm0.005$ $\lambda_2=0.076\pm0.005$ $\lambda_3=-0.021\pm0.005$ $\lambda_4=-0.77\pm0.03$
Lyapunov Dimensions	$D=2.1\pm0.2$	$D=3.3\pm0.1$

increasing dimension of the embedding phase space [13]. The results computed for the two characteristic data files are listed in Table 1. Here we have used embeddings of dimension from 20 to 26 (cf. Fig. 5 of [11]). We conclude that the states A and B manifest different strange attractors, the chaotic behavior of the second one reflecting a considerably higher degree of freedom. The closeness of $D(0)$ and $D(1)$ as well as $K(0)$ and $K(1)$ indicates an almost self-similar structure for both chaotic attractors, not yet being adequate to confirm multifractal behavior of the system. In order to yield a more complete characterization, we looked at the spectra of invariant static scaling indices $f(\alpha)$, describing the global distribution of singularities on a fractal measure [14]. Their graphs are given in Fig. 3 for the two data files considered. It is now clearly seen that both chaotic attractors display multifractal structures, the extension and the form of which are changed drastically between the two working conditions. Moreover, the numerical values of the generalized fractal dimensions are fairly well reproduced by the corresponding scaling functions.

Finally, we have evaluated the generalized Lyapunov exponents λ_i together with the corresponding spectra of invariant dynamical scaling indices $\Phi(\Lambda)$ using the algorithm developed by Stoop and Meier [15]. The Lyapunov characteristic exponents were estimated from the linearized dynamics constructed by a least-squares fit, based on a modified and improved version of the proposals put forward by Eckmann et al. [16] and Sano et al. [13, 17]. As summarized in Table 1, we detected three (four) relevant exponents from data file A (B) for embeddings of dimension from 7 to 10 (8 to 11). In accordance with the gradually increasing dimensionality, the two chaotic states are further discriminated by a different number of positive Lyapunov exponents, determining mutually independent directions of stretching and folding-over of nearby trajectories in phase space and, thus, reflecting the order of chaos [15–18]. Adopting the terminology introduced by Rössler [4, 19], the first state is called "ordinary chaotic" (three-variable chaos defined by one positive exponent), the higher-order analogue of the second state "hyperchaotic" (four-variable chaos defined by two positive exponents). The global spectral characterization of these chaotic dynamics can be deduced from the scaling functions for the generalized Lyapunov exponents [20]. Their graphs in Fig. 4 show the different dynamical complexity of the attractors. The spreading and the shifted

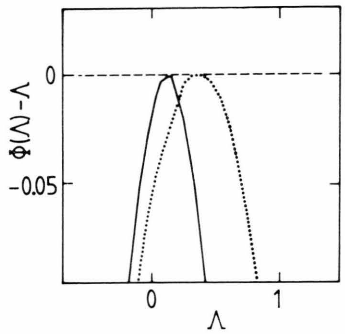

Fig. 4. Dynamical scaling functions of different chaotic attractors calculated from the data files A (solid curve) and B (dotted curve).

position of the $\Phi(\Lambda)$ spectra are qualitatively seen in $f(\alpha)$. Again, the numerical values of the generalized entropies can be reproduced by the corresponding dynamical scaling functions [11].

One conjecture that unifies probabilistic and dynamical properties of an attracting set is the Kaplan-Yorke relationship [21]. Therefore, we derived from the Lyapunov spectrum the corresponding dimension

$$D = j + \sum_{i=1}^{j} \lambda_i / |\lambda_{j+1}|,$$

where j is defined by the condition that

$$\sum_{i=1}^{j} \lambda_i > 0 \quad \text{and} \quad \sum_{i=1}^{j+1} \lambda_i < 0.$$

The results obtained for the two chaotic attractors are given in Table 1. Comparison between the Lyapunov dimension D and the information dimension $D(1)$ calculated independently shows satisfactory agreement within experimental accuracy of one standard deviation. A further conjecture predicts that the Kolmogorov-Sinai entropy $K(1)$ corresponds to the lower bound of the sum of all positive Lyapunov exponents [16, 22]. The apparent closeness of these quantities in our experiment (in contrast to other dynamical systems [23]) may indicate the manifestation of peculiar strange attractors, occurring near quasiperiodic flows on an m-torus ($m \geqq 3$) that are governed by horseshoe-like diffeomorphisms (Newhouse-Ruelle-Takens theorem) [4, 24]. We suspect that the chaotic hierarchy inherent to the present semiconductor system is generated by *weak* nonlinear coupling of spatially localized oscillatory subsystems. Indeed, we have found that the evaluation of dimensions, entropies, Lyapunov exponents, and corresponding scaling functions for different local voltage drops V_i along the sample yields nearly identical results – in accordance to earlier conjectures [3, 8].

To conclude, an exemplary semiconductor system is shown to undergo different degrees of chaotic behavior. By the help of distinct numerical analysis procedures a self-consistent picture of the physical situation investigated is obtained. This picture fits well into the model of a multicomponent reaction-diffusion system, capable of generating a weak chaotic hierarchy of near-quasiperiodic strange attractors. From the standpoint of coupled logistic maps just being studied to mimic the sine-Gordon equation, one might speculate that an universal scaling law should be observed on the ladder towards higher chaos, analogous to that discovered by Feigenbaum for the successive, ever closer-spaced appearance of higher and higher periodic solutions ending in chaos.

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